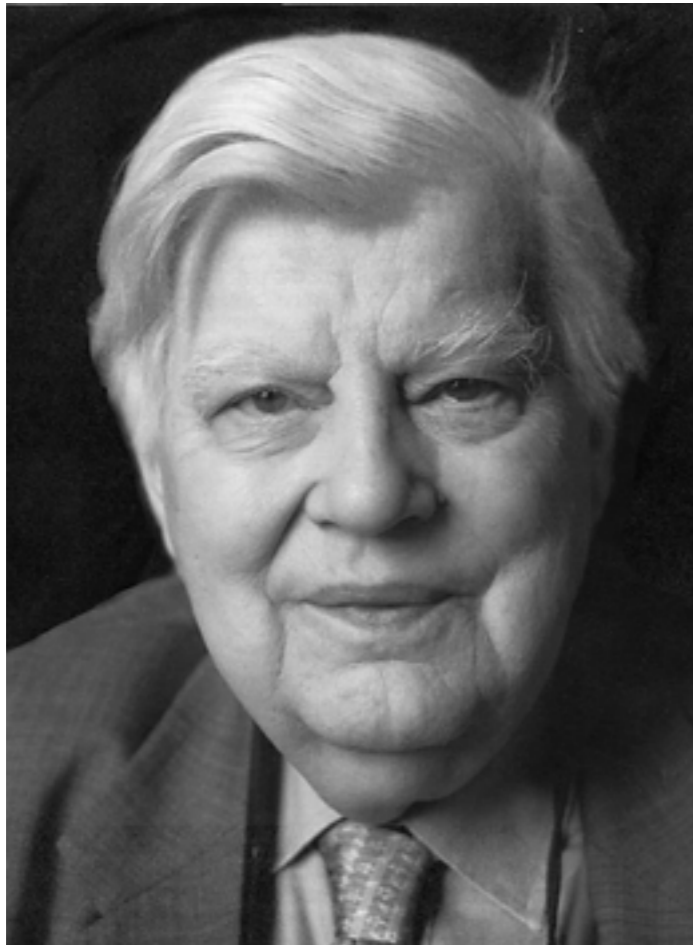


Usefulness of Deduction and Proof-theoretic semantics

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- Dummett: validity and usefulness
- Bilateralist conception of consequence

Justification of Deduction (1973)



M. Dummett

... an explanation, not of why we should accept certain forms of argument ... but of *how deductive argument is possible at all*.

(297; my emphasis)

The tension

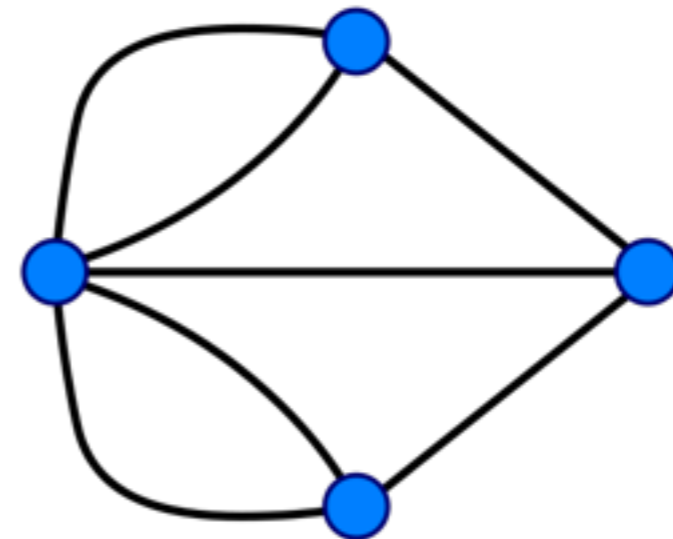
- Validity
- Usefulness

Seven bridges of Königsberg



each bridge once and only once?

Seven bridges of Königsberg



No

Someone has crossed every bridge

She has crossed some bridge at least twice

- Useful: knowledge of the conclusion *without* a direct observation
- Valid: the conclusion is true whenever the premise is true, hence convincing



Direct and indirect means
of establishing truth

Direct verification

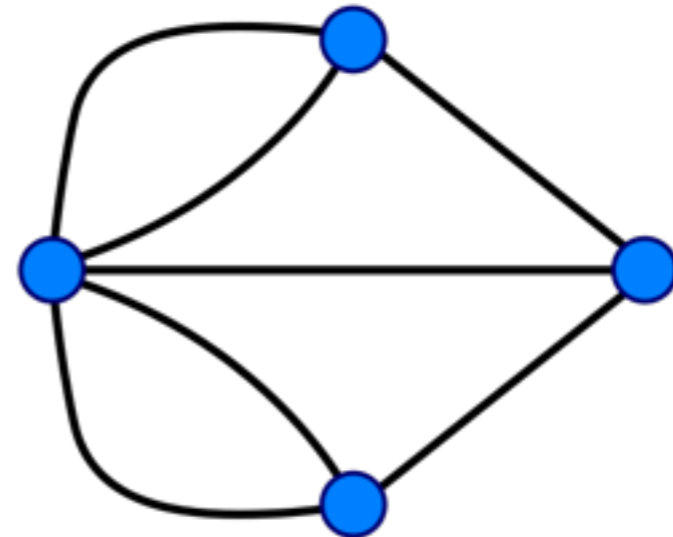
$$\frac{\begin{array}{cc} \downarrow & \downarrow \\ p & q \end{array}}{p \wedge q} \\ \frac{}{(p \wedge q) \vee r}$$

A process corresponding to
the internal structure

in accordance with its
meaning
compositionally explained

Someone has crossed every bridge

She has crossed some bridge at least twice



without any *direct observation*

Indirect verification

$$\frac{A \rightarrow B \quad \vdots \quad A}{B}$$

unbound complexity

Conservative Extension

The sum of the digits of 732 is a multiple of 3

732 itself is a multiple of 3

Let $n = 100a + 10b + c$

Assume: $a + b + c = 3k$ for some k

$$\begin{aligned}n &= 100a + 10b + c \\&= 99a + 9b + a + b + c \\&= 99a + 9b + 3k \\&= 3(33a + 3b + k)\end{aligned}$$

direct verification
of the premise



direct verification
of the conclusion

effective method

$$\frac{A_0 \wedge A_1}{A_i} (\wedge E)$$

$$\frac{\frac{\frac{\Pi_0}{A_0} \quad \frac{\Pi_1}{A_1}}{A_0 \wedge A_1} (\wedge I)}{A_i} (\wedge E) \quad \vdash \quad \frac{\Pi_i}{A_i}$$

direct verification
of the premise



direct verification
of the conclusion

effective method

Validity of indirect inference

direct verification
of the premise



direct verification
of the conclusion

effective method

Conservativity

- No new consequence which could not already be established without the inference
- A.k.a. *Harmony*

Beyond direct
verification

Case 1. multiple of 3

- Seems easily reduced to direct computation
- How about a larger input, or a far more complicated computation or proof ?
- Reduction procedure would remain effective *in principle*, but not *in practice*

Case 2. Königsberg

direct verification
of the premise



direct verification
of the conclusion

Euler's proof

Direct observation may
no longer be accessible

[A]n effective method for arriving at a direct verification of the statement, provided that we are given a *sufficiently detailed set of observations*

(314)

Such [indirect] inferences will lead us to conclusions at which, in *actual circumstances*, we could not have arrived without the employment of those mode of reasoning.

(316)

Truth

- We accept the conclusion as true though (direct) verification is impossible in practice
- Usefulness comes with some notion of truth transcending (direct) verification

The chain of justification comes to an end: we simply do. Not only our language, but our entire conception of the world, would be transformed if we did not...

Logical Basis of Metaphysics, 179

Verificationist meaning theory

Proof-Theoretic Semantics

- Meaning determined in terms of direct verification (canonical proof)
- Justification of indirect inference based on conservativity or harmony
- Appropriate notion of truth transcending direct verification

Bilateralist conception of
validity and usefulness

direct
verification

effectiv
in practice

The truth could have been recognized
by a being with sufficient perceptual
and intellectual powers?

No longer
accessible

...

The truth *could have* been recognized
given sufficiently detailed observation

verification

falsification

cf. Restall, Meaning, rules, and
defining concepts.

Direct
verification



$$A \vdash B$$



Indirect
verification

Direct
verification



$$A \vdash B$$



Exclusion of
falsification

$$A \vdash B$$

- A verification of A and falsification of B involves a *clash*
- A verification of A excludes any falsification of B
- A falsification of B excludes any verification of A
- cf. to assert A and to deny B is to make a mistake (Restall, 2005)

$$A \vdash A \text{ (id)}$$

A verification and a falsification
of A clash with each other

Derivation in sequent system

$$\begin{array}{c}
 \frac{p \vdash p}{p \vdash p \vee q} \quad (\vee R) \\
 \frac{p \vdash p \vee q}{p, \neg(p \vee q) \vdash} \quad (\neg L) \\
 \frac{p, \neg(p \vee q) \vdash}{\neg(p \vee q) \vdash \neg p} \quad (\neg R) \\
 \hline
 \neg(p \vee q) \vdash \neg p \wedge \neg q
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{q \vdash q}{q \vdash p \vee q} \quad (\vee R) \\
 \frac{q \vdash p \vee q}{q, \neg(p \vee q) \vdash} \quad (\neg L) \\
 \frac{q, \neg(p \vee q) \vdash}{\neg(p \vee q) \vdash \neg q} \quad (\neg R) \\
 \hline
 \neg(p \vee q) \vdash \neg q \quad (\wedge R)
 \end{array}$$

showing the way a “complex” clash
is made up from simpler ones

verification

Identity

No overlap

falsification

verification

falsification

verification

falsification

(Exclusion of) falsification
explains usefulness of deduction

...and allows us to circumscribe
truth from the outside

Validity

$$\frac{A \vdash C \quad C \vdash B}{A \vdash B} \text{ (Cut)}$$

An indirect verification of C
can be used as a verification of C

Admissibility of Cut ensures faithfulness

Cut elimination

$$\frac{\frac{\frac{\Pi_0}{X \vdash A} \quad \frac{\Pi_1}{X \vdash B}}{X \vdash A \wedge B} \quad \frac{\frac{\Pi}{A \vdash C}}{A \wedge B \vdash C}}{X \vdash C} \text{ (Cut)}}{\quad \mapsto \quad \frac{\frac{\Pi_0}{X \vdash A} \quad \frac{\Pi}{A \vdash C}}{X \vdash C} \text{ (Cut)}}$$

Showing how indirect verification is coordinated with direct verification

$$\frac{A \vdash C \quad C \vdash B}{A \vdash B} \text{ (Cut)}$$

- A verification of A excludes any falsification of C
- A falsification of B excludes any verification of C
- Such a combination of a verification of A and a falsification of B involves a clash

$$\frac{A \vdash C \quad C \vdash B}{A \vdash B} \text{ (Cut)}$$

- No gap between verifiability and falsifiability
- Not: there is either a verification or a falsification
- Exclusion of falsification (verification) does not mean actual existence of verification (falsification)

verification

Coordinated
through Cut



falsification

verification

Coordinated
through Cut



falsification

verification

Cut

No gap

falsification

- Dummett: Conservative extension to truth-recognition in hypothetical settings
- Bilateralist: Coordination between verification and falsification