Logical Constants and Focusing Disagreements

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Plan

Motivation Defining Rules Disagreement Booleans Quantifiers Modals Limits

MOTIVATION

Expressivism

Logical vocabulary increases our expressive capacity.

Subject Matter Independence

Logical vocabulary is subject matter independent.

How do these fit together?

What distinctive things can we *do* with logical vocabulary?

DEFINING RULES

Sequent Calculus

$$X \vdash Y$$

Don't assert X and deny Y.

Rules

$$\frac{X,A \vdash Y}{\overline{X \vdash \neg A,Y}}$$

$$\frac{X,A,B \vdash Y}{\overline{X,A \land B \vdash Y}}$$

$$\frac{X,A \vdash B,Y}{\overline{X \vdash A \to B,Y}}$$

$$\frac{X \vdash A, B, Y}{\overline{X \vdash A \lor B, Y}}$$

DISAGREEMENT

Hinges

... the questions that we raise and our doubts depend upon the fact that some propositions are exempt from doubt, are as it were like hinges on which those turn.

That is to say, it belongs to the logic of our scientific investigations that certain things are in deed not doubted. But it isn't that the situation is like this: We just can't investigate everything, and for that reason we are forced to rest content with assumption.

If I want the door to turn, the hinges must stay put.

—Ludwig Wittgenstein, On Certainty §341–3

Not *Propositions*, but *Vocabulary*

Logical vocabulary can function as fixed points, around which disagreement can take place.

 $p \rightarrow q$, p. Therefore, q

$p \rightarrow q$, p. Therefore, q

Accept $p \rightarrow q$, accept p, accept qReject $p \rightarrow q$, accept p, accept qAccept $p \rightarrow q$, reject p, accept qReject $p \rightarrow q$, reject p, accept qAccept $p \rightarrow q$, accept p, reject qReject $p \rightarrow q$, accept p, reject qAccept $p \rightarrow q$, reject p, reject qReject $p \rightarrow q$, reject p, reject q

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That looks like truth tables.

Can we look at this proof theoretically?

Taking Positions

assert p	deny p
accept p	reject p
	—р

Taking Positions

assert p	disclaim p	deny p
accept p	be open about p	reject p
	?p	-р

How to understand '?'

Think of ?p as being *open* to both +p and -p.

(That works for the state, not the speech act.)

Positions

A POSITION is a collection of positively and negatively tagged statements.

Disagreement and Distinction

- ► Two positions *disagree* when one contains +p and the other −p for some p.
- ► Two positions are *distinct* when one contains a positively or negatively tagged formula and the other doesn't.

Slogan

Distinct positions on logically complex statements can be focused onto distinct positions on their constituents.

BOOLEANS

Abelard:
$$+(A \land B)$$
 Eloise: $-(A \land B)$

Example—Abelard

$$\frac{X,A,B\vdash Y}{\overline{X,A\land B\vdash Y}}$$

$$A \wedge B \vdash A$$
 $A \wedge B \vdash B$

Abelard cannot coherently add -A or -B. So he's not (coherently) open to -A and -B. He's committed (implicitly) to +A and to +B.

Example—Eloise

$$\frac{X, A, B \vdash Y}{\overline{X, A \land B \vdash Y}}$$

$$A, B \vdash A \land B$$

Eloise cannot coherently add +A together with +B. So she's not (*coherently*) open to +A together with +B.

> But she can be open to +A. And she can be open to +B too.

Example (cont.)

Abelard	Eloise
	-A, -B
+A, +B	+A, -B
	-A, +B

What about $?(A \land B)$

Abelard:
$$?(A \land B)$$
 Eloise: $-(A \land B)$

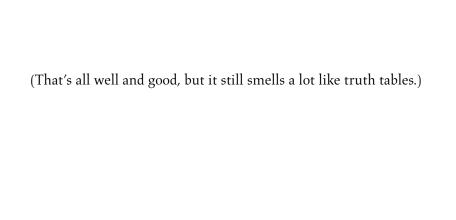
Then Abelard is *open* to +A, +B and Eloise isn't, so there is still a difference concerning A and B.

All the Booleans

$$\frac{X, A \vdash Y}{X \vdash \neg A, Y} \qquad \frac{X, A, B \vdash Y}{X, A \land B \vdash Y}$$

$$\frac{X, A \vdash B, Y}{X \vdash A \to B, Y} \qquad \frac{X \vdash A, B, Y}{X \vdash A \lor B, Y}$$

ONCE WE FIX THESE RULES, difference concerning negations, conjunctions, disjunctions, and conditionals can be focused into difference concerning their *constituents*.



QUANTIFIERS

Quantifiers

$$\frac{X \vdash A|_{y}^{x}, Y}{X \vdash \forall x \, A, Y} \qquad \frac{X, A|_{y}^{x} \vdash Y}{X, \exists x \, A \vdash Y}$$

(where y is not free in X and Y)

Abelard and Eloise again

Abelard: $+\exists x A$ Eloise: $-\exists x A$

Example—Eloise

$$\frac{X, A|_{y}^{x} \vdash Y}{\overline{X, \exists x A \vdash Y}}$$

$$A|_y^x \vdash \exists x A$$

Eloise cannot coherently add $+A|_{y}^{x}$ for any y at all. So she's not (coherently) open to $+A|_{y}^{x}$.

So she's (*implicitly*) committed to $-A|_y^x$ for any y whatever.

Example—Abelard

$$\frac{X, A|_{y}^{x} \vdash Y}{X, \exists x A \vdash Y}$$

$$\frac{X, A|_{y}^{x}, \exists x A \vdash Y}{X, \exists x A \vdash Y}$$

Abelard can add $+A|_y^x$ (for a new y) at no cost to coherence.

Why? To simplify—to expose the strucure of A.

In Dialogue...

If I assert $\exists x A$, you can ask me to introduce a new term y, and in that context I'm committed to $+A|_{u}^{x}$.

"You say $\exists x A$. Suppose that's right. Call it y. So $A|_y^x \dots$ "

In Dialogue...

If I assert $\exists x A$, you can ask me to introduce a new term y, and in that context I'm committed to $+A|_y^x$.

"You say $\exists x A$. Suppose that's right. Call it y. So $A|_y^x \dots$ "

Differences over quantifiers

Differences over $\exists x \ A \ (\text{or} \ \forall x \ A)$ can become differences over A, at the cost of new vocabulary.

MODALS

Differences over quantifiers

Differences over $\Diamond A$ (or $\Box A$) can become differences over A, at the cost of new *zones*.

In Dialogue...

If I assert $\lozenge A$, you can ask me to introduce a new zone and in that context I'm committed to +A.

"You say ◊A. Suppose that's right. Consider if that obtained. Then, A . . . '

In Dialogue...

If I assert $\lozenge A$, you can ask me to introduce a new zone and in that context I'm committed to +A.

"You say $\Diamond A$. Suppose that's right. Consider if that obtained. Then, $A \dots$ "

Rules

$$\frac{\mathcal{H}[A \vdash \mid X \vdash Y]}{\mathcal{H}[X, \lozenge A \vdash Y]}$$

Example

$$\frac{ \begin{array}{c} p \vdash p \mid \vdash \Diamond q \\ \hline p \vdash \mid \vdash \Diamond p, \Diamond q \end{array} \quad \begin{array}{c} q \vdash q \mid \vdash \Diamond p \\ \hline q \vdash \mid \vdash \Diamond p, \Diamond q \\ \hline \\ \hline \\ \frac{p \lor q \vdash \mid \vdash \Diamond p, \Diamond q}{\Diamond (p \lor q) \vdash \Diamond p, \Diamond q} \\ \hline \\ \hline \\ \Diamond (p \lor q) \vdash \Diamond p \lor \Diamond q \\ \hline \end{array}}$$

Of course...

 \dots sense needs to be made of +A and -A in different zones.

LIMITS

Enough of where this can work.

Where does it *fail*?

Predicates

Differences concerning Fa needn't focus...

Identity?

 \dots including differences over a=b \dots

Identity?

... unless we are *very* generous about what counts as a constituent.

Identity?

Abelard:
$$+(a = b)$$
 Eloise: $-(a = b)$

Identity—Abelard

$$a = b, Fa \vdash Fb$$

Abelard is barred from $+C|_{a}^{x}$ with $-C|_{b}^{x}$.

Identity—Eloise

But Eloise?

If Eloise denies a = b, Abelard can ask her to introduce a new predicate X and she is committed to +Xa and to -Xb.

"You deny a = b. Suppose they're different..."

Identity—Eloise

But Eloise?

If Eloise denies a = b, Abelard can ask her to introduce a new predicate X and she is committed to +Xa and to -Xb.

"You deny a = b. Suppose they're different..."

Constituents, subformulas...

This is *stretching* the notion of a constituent (or a subformula)...

... but it's still wellfounded.

Second Order Quantifiers?

Abelard: $+\forall X A$ Eloise: $-\forall X A$

Second Order Quantifiers—Abelard

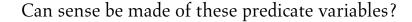
$$\forall X A \vdash A|_{\lambda x.B}^{X}$$

Abelard is barred from $-A|_{\lambda x.B}^{X}$.

Second Order Quantifiers—Eloise

But Eloise?

If Eloise denies $\forall X A$, then Abelard can ask her to introduce a new predicate Y and she is committed to $-A|_Y^X$.



Reductions

(The formulas shrink. The transitive closure of → is well founded.)

Instances

We do not reduce $\forall X A$ to *each* of its instances. That relation is not well founded.

$$\forall X\,X\alpha>X\alpha|_{\lambda x.\forall X\,X\alpha}^{X}=\forall X\,X\alpha$$

Differences over $\forall X A$ resolve into differences over its free variable instances.

Expressive gain, no new subject matter

- ► GAIN: we can say new things with our new vocabulary.
- ► MODESTY: if we had *new* subject matter, we could differ on that, keeping our other commitments fixed.
 - What about defined terms? Suppose we treat 'bachelor' as 'unmarried male'? Can we disagree about whether something is a bachelor without disagreeing about 'unmarried' and 'male'?
 - What do we treat as a subformula in this case?

THANK YOU!