Logical Constants and Focusing Disagreements

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Motivation

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MOTIVATION

Logical vocabulary increases our expressive capacity.

Logical vocabulary is subject matter independent.

How do these fit together?

What distinctive things
can we *do* with logical vocabulary?

DEFINING RULES

$$X \vdash Y$$

Don't assert X and deny Y.

Rules

$$\frac{X, A \vdash Y}{X \vdash \neg A, Y}$$

$$\frac{X, A, B \vdash Y}{X, A \land B \vdash Y}$$

$$\frac{X, A \vdash B, Y}{X \vdash A \to B, Y}$$

$$\frac{X \vdash A, B, Y}{X \vdash A \lor B, Y}$$

DISAGREEMENT

… the questions that we raise and our doubts depend upon the fact that some propositions are exempt from doubt, are as it were like hinges on which those turn.

That is to say, it belongs to the logic of our scientific investigations that certain things are in deed not doubted. But it isn't that the situation is like this: We just can't investigate everything, and for that reason we are forced to rest content with assumption.

If I want the door to turn, the hinges must stay put.

—Ludwig Wittgenstein, *On Certainty* §341–3

Logical vocabulary can function as fixed points, *around which* disagreement can take place.

$p \to q$, p. Therefore, q

Example

$$p \to q, p. \text{ Therefore, } q$$

Accept $p \to q$, accept p, accept q

Reject $p \to q$, accept p, accept q

Accept $p \to q$, reject p, accept q

Reject $p \to q$, reject p, accept q

Accept $p \to q$, accept p, reject q

Reject $p \to q$, accept p, reject q

Accept $p \to q$, reject p, reject q

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~~Accept $p \to q$, accept p, reject q~~

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Accept $p \to q$, reject p, reject q

Reject $p \to q$, reject p, reject q

$p \to q$, p. Therefore, q

Accept $p \to q$, accept p, accept q

~~Reject $p \to q$, accept p, accept q~~

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~~Accept $p \to q$, accept p, reject q~~

Reject $p \to q$, accept p, reject q

Accept $p \to q$, reject p, reject q

~~Reject $p \to q$, reject p, reject q~~

That looks like truth tables.

Can we look at this proof theoretically?

assert p	deny p
accept p	reject p
$+p$	$-p$

assert p	*disclaim* p	deny p
accept p	*be open about* p	reject p
$+$p	?p	$-$p

Think of ?p as being *open* to both +p and −p.

(That works for the state, not the speech act.)

A POSITION is a collection of positively and negatively tagged statements.

- Two positions *disagree* when one contains $+p$ and the other $-p$ for some p.

- Two positions are *distinct* when one contains a positively or negatively tagged formula and the other doesn't.

Distinct positions on logically complex statements can be focused onto distinct positions on their constituents.

BOOLEANS

Abelard: $+(A \wedge B)$ Eloise: $-(A \wedge B)$

$$\frac{X, A, B \vdash Y}{X, A \land B \vdash Y}$$

$$A \land B \vdash A \qquad A \land B \vdash B$$

Abelard cannot coherently add $-A$ or $-B$.
So he's not (*coherently*) open to $-A$ and $-B$.
He's committed (*implicitly*) to $+A$ and to $+B$.

$$\frac{X, A, B \vdash Y}{X, A \land B \vdash Y}$$

$$A, B \vdash A \land B$$

Eloise cannot coherently add $+A$ together with $+B$.
So she's not (*coherently*) open to $+A$ together with $+B$.

But she can be open to $+A$.
And she can be open to $+B$ too.

Abelard	*Eloise*
	$-A, -B$
$+A, +B$	$+A, -B$
	$-A, +B$

Abelard: $?(A \wedge B)$ Eloise: $-(A \wedge B)$

Then Abelard is *open to* $+A, +B$ and Eloise isn't, so there is still a difference concerning A and B.

$$\frac{X, A \vdash Y}{\overline{X \vdash \neg A, Y}} \qquad \frac{X, A, B \vdash Y}{\overline{X, A \land B \vdash Y}}$$

$$\frac{X, A \vdash B, Y}{\overline{X \vdash A \to B, Y}} \qquad \frac{X \vdash A, B, Y}{\overline{X \vdash A \lor B, Y}}$$

ONCE WE FIX THESE RULES, difference concerning negations, conjunctions, disjunctions, and conditionals can be focused into difference concerning their *constituents*.

(That's all well and good, but it still smells a lot like truth tables.)

QUANTIFIERS

$$\frac{X \vdash A|^x_y, Y}{X \vdash \forall x\, A, Y} \qquad \frac{X, A|^x_y \vdash Y}{X, \exists x\, A \vdash Y}$$

(where y is not free in X and Y)

Abelard: $+\exists x\, A$ Eloise: $-\exists x\, A$

$$\frac{X, A|^x_y \vdash Y}{X, \exists x\, A \vdash Y}$$

$$A|^x_y \vdash \exists x\, A$$

Eloise cannot coherently add $+A|^x_y$ for any y at all.
So she's not (*coherently*) open to $+A|^x_y$.

So she's (*implicitly*) committed to $-A|^x_y$ for any y whatever.

$$\frac{X, A|^x_y \vdash Y}{X, \exists x\, A \vdash Y}$$

$$\frac{X, A|^x_y, \exists x\, A \vdash Y}{X, \exists x\, A \vdash Y}$$

Abelard *can* add $+A|^x_y$ (for a new y) at no cost to coherence.

Why? To simplify—to expose the strucure of A.

If I assert $\exists x\, A$,
you can ask me to introduce a new term y,
and in that context I'm committed to $+A|^x_y$.

"You say $\exists x\, A$. Suppose that's right.
Call it y. So $A|^x_y$ …"

If I assert $\exists x\, A$,
you can ask me to introduce a new term y,
and in that context I'm committed to $+A|^x_y$.

"You say $\exists x\, A$. Suppose that's right.
Call it y. So $A|^x_y$ …"

Differences over $\exists x\, A$ (or $\forall x\, A$)
can become differences over A,
at the cost of new vocabulary.

MODALS

Differences over $\Diamond A$ (or $\Box A$)
can become differences over A,
at the cost of new *zones*.

If I assert $\Diamond A$,
you can ask me to introduce a new zone
and in that context I'm committed to $+A$.

"You say $\Diamond A$. Suppose that's right.
Consider if that obtained. Then, A …"

If I assert $\Diamond A$,
you can ask me to introduce a new zone
and in that context I'm committed to $+A$.

"You say $\Diamond A$. Suppose that's right.
Consider if that obtained. Then, A …"

$$\frac{\mathcal{H}[A \vdash \ \ \mid X \vdash Y]}{\mathcal{H}[X, \Diamond A \vdash Y]}$$

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{p \vdash p \mid\ \vdash \Diamond q}{p \vdash\ \mid\ \vdash \Diamond p, \Diamond q} \qquad \dfrac{q \vdash q \mid\ \vdash \Diamond p}{q \vdash\ \mid\ \vdash \Diamond p, \Diamond q}}{p \vee q \vdash\ \mid\ \vdash \Diamond p, \Diamond q}}{\Diamond(p \vee q) \vdash \Diamond p, \Diamond q}}{\Diamond(p \vee q) \vdash \Diamond p \vee \Diamond q}$$

… sense needs to be made of $+A$ and $-A$ in different zones.

LIMITS

Enough of where this can work.

Where does it *fail*?

Differences concerning Fa needn't focus…

… including differences over $a = b$ …

... unless we are *very* generous
about what counts as a constituent.

Abelard: $+(a = b)$ Eloise: $-(a = b)$

$$a = b, Fa \vdash Fb$$

Abelard is barred from
$+C|^x_a$ with $-C|^x_b$.

But Eloise?

If Eloise denies $a = b$,
Abelard can ask her to introduce a new predicate X
and she is committed to $+Xa$ and to $-Xb$.

"You deny $a = b$. Suppose they're different…"

But Eloise?

If Eloise denies $a = b$,
Abelard can ask her to introduce a new predicate X
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"You deny $a = b$. Suppose they're different…"

This is *stretching* the notion of a constituent (or a subformula)…

… but it's still wellfounded.

Abelard: $+\forall X\, A$ Eloise: $-\forall X\, A$

$$\forall X\, A \vdash A|^{X}_{\lambda x.B}$$

Abelard is barred from $-A|^{X}_{\lambda x.B}$.

But Eloise?

If Eloise denies $\forall X\, A$,
then Abelard can ask her
to introduce a new predicate Y
and she is committed to $-A|^{X}_{Y}$.

Can sense be made of these predicate variables?

$$
\begin{array}{rcl}
\hline
\neg A & \rightsquigarrow & A \\
A \wedge B & \rightsquigarrow & A, B \\
A \vee B & \rightsquigarrow & A, B \\
A \rightarrow B & \rightsquigarrow & A, B \\
\hline
\forall x\, A & \rightsquigarrow & A|^{x}_{x_0}, A|^{x}_{x_1}, A|^{x}_{x_2}, \ldots \\
\exists x\, A & \rightsquigarrow & A|^{x}_{x_0}, A|^{x}_{x_1}, A|^{x}_{x_2}, \ldots \\
\hline
a = b & \rightsquigarrow & X_0\, a, X_0\, b, X_1\, a, X_1\, b, X_2\, a, X_2\, b, \ldots \\
\hline
\forall X\, A & \rightsquigarrow & A|^{X}_{X_0}, A|^{X}_{X_1}, A|^{X}_{X_2}, \ldots \\
\exists X\, A & \rightsquigarrow & A|^{X}_{X_0}, A|^{X}_{X_1}, A|^{X}_{X_2}, \ldots \\
\hline
\end{array}
$$

(The formulas shrink. The transitive closure of $\rightsquigarrow$ is well founded.)

We do not reduce $\forall X\, A$ to *each* of its instances.
That relation is not well founded.

$$\forall X\, X a > X a|^{X}_{\lambda x. \forall X\, X a} = \forall X\, X a$$

Differences over $\forall X\, A$ resolve into
differences over its free variable instances.

Expressive gain, no new subject matter

- GAIN: we can say new things with our new vocabulary.
- MODESTY: if we had *new* subject matter, we could differ on that, keeping our other commitments fixed.
  - What about *defined terms*? Suppose we treat '*bachelor*' as '*unmarried male*'? Can we disagree about whether something is a bachelor without disagreeing about 'unmarried' and 'male'?
  - What do we treat as a subformula in this case?

THANK YOU!